Int. J. Heat Mass Transfer. Vol. 19. pp. 1273–1280. Pergamon Press 1976. Printed in Great Britain

COMBINED BODY FORCE AND FORCED CONVECTION IN LAMINAR FILM CONDENSATION OF MIXED VAPOURS—INTEGRAL AND FINITE DIFFERENCE TREATMENT

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(*Received* 5 *December* 1975)

Abstract—Laminar film condensation at combined body force and forced convection of a binary vapour at a flat plate is considered. After a suitable coordinate transformation the general solution is seen to be structured according to a dimensionless length ξ. The system of equations is solved by a finite difference method and two integral methods. Both integral treatments differ from each other by the expression for the velocity profile, one admitting an inflection point. This more complicated expression for the velocity profile is important, when the case of low condensation rate and dominating influence of body force is treated. Generally speaking, the integral treatment is very satisfactory. The influence of heat transfer in the vapour, variable fluid properties and thermodynamic coupling by thermal diffusion and diffusional thermo is discussed.

NOMENCLATURE

c_p, heat capacity;
D, diffusion coefficient;
g^*, gravity in direction of flow;
h, enthalpy;
Δh_v, enthalpy of evaporation;
j, diffusional flux;
$\dot{m}$, mass flux;
$\tilde{M}$, molar weight;
Nu, Nusselt number;
Re_{Lx}, $= \frac{u_\infty x}{\nu_L}$, Reynolds number with kinematic viscosity of liquid;
T, temperature;
u, velocity in x-direction;
v, velocity in y-direction;
x, coordinate along the plate;
x_1, mass species concentration of methanol in the liquid;
y, coordinate perpendicular to the plate;
y_1, mass species concentration of methanol in the vapour.

Greek symbols

α_T, thermal diffusion factor;
δ, thickness of velocity boundary layer;
δ_L, thickness of condensate film;
δ_{y_1}, thickness of species layer;
η, dimensionless coordinate in the vapour;
λ, heat conductivity;
μ, dynamical viscosity;
ν, kinematic viscosity;
ξ, $= \frac{g^* x}{u_\infty^2}$, dimensionless coordinate;
ρ, density;
τ, shear force.

Subscripts

i, at the interface;
L, in the condensate;
∞, in the free stream;
w, at the wall.

INTRODUCTION

IN THE last ten years, considerable work has been done on film condensation of mixed vapours using boundary-layer theory. Sparrow and Lin [1], Minkowycz and Sparrow [2], Sparrow, Minkowycz and Saddy [3], Denny, Mills and Jusionis [4], Denny and Jusionis [5] and Jones and Renz [6] solved the equations for various cases of film condensation for mixtures of vapours and non-condensable gases. Koh and Grafton [7], Koh [8], Sparrow and Marschall [9], Marschall and Hickmann [10], Taitel and Tamir [11], Denny and South [12], Denny and Jusionis [13] and Taitel, Tamir and Schlünder [14] reported solutions on similar cases for mixtures of condensable vapours. The present paper gives an overall solution of combined body force and forced convection in film condensation of mixed vapors which contains so far solved problems as limiting cases. It investigates in particular the merits of an exact finite difference treatment and its inherent difficulties in comparison with two formulations of the approximate integral technique. The transition of the general problem to the asymptotic cases of pure body force and forced convection is studied in some detail as it is important in practical application. The influence of heat transfer in the vapour, variable thermophysical properties and thermodynamic coupling is discussed in connection with the objective of reducing computational effort.

THE PHYSICAL MODEL

A flat plate with arbitrary inclination, measured by α, is considered, Fig. 1. The gravity has the component

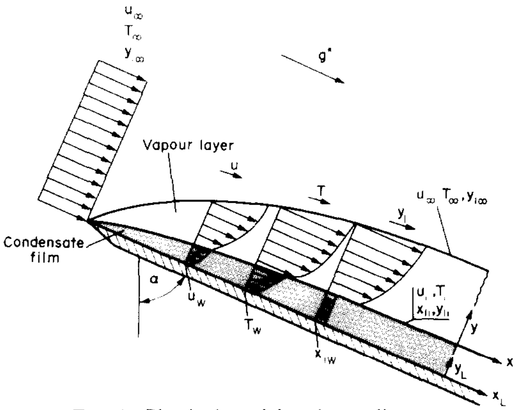

FIG. 1. Physical model and coordinates.

g^* in the direction of flow. The vapour has a free stream velocity of u_∞, its temperature is T_∞. Under the condition of saturation at T_∞, the concentration $y_{1\infty}$ of the vapour is defined. The condensate is assumed to form a laminar, wavefree film on the plate. The temperature at the film surface is identical in liquid and vapour, i.e. no thermal resistance is assumed to exist at the interface. The concentrations in vapour and liquid at the interface are determined by the condition of thermodynamic equilibrium. Complete solubility is assumed in the liquid phase. The vapour flow is assumed to be laminar, too. The speculation by Shekriladse and Gomelauri [15] that this should in practical cases be the case because of the stabilizing effect that suction has on the vapour layer has become questionable by the work of Jones and Renz [6], who calculate turbulent profiles on the basis of experimental results. Here, this assumption is part of the physical model.

THE ANALYTICAL MODEL

In principle the analytical treatment should start with the full conservation equations. Specific features of the problem, however, allow a few simplifications of great importance. For the liquid film, it is generally accepted that Nusselt's assumptions may be used for non-metallic liquids at technical conditions, when the thermophysical properties are evaluated at a suitable reference condition, for which is chosen here a temperature

$$T_{Lr} = T_i + \tfrac{1}{3}(T_i - T_w) \tag{1}$$

and the species concentration x_{1i} of the saturated liquid at T_i. Comments on such a procedure can be found in the papers by Poots and Miles [16] and Denny and Mills [17]. The simplified equations for the condensate film are

$$\mu_L \frac{\partial^2 u_L}{\partial y_L^2} + g^*(\rho - \rho_\infty) = 0 \tag{2}$$

$$\frac{\partial^2 T}{\partial y_L^2} = 0 \tag{3}$$

$$\frac{\partial^2 x_1}{\partial y_L^2} = 0. \tag{4}$$

Equation (4) implies that there is no significant resistance to mass transfer in the liquid. It predicts a uniform concentration profile in the film. This of course also neglects that the local concentration of the film is influenced by oncoming liquid from above the location considered. For the results presented in this paper, this is immaterial as shown in [13].

For the vapour flow, Prandtl's boundary-layer assumptions are introduced to give

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{5}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial \tau}{\partial y} + g^*(\rho - \rho_\infty) \tag{6}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = -\frac{\partial q}{\partial y} - j\frac{\partial}{\partial y}(h_1 - h_2) \tag{7}$$

$$\rho\left(u\frac{\partial y_1}{\partial x} + v\frac{\partial y_1}{\partial y}\right) = -\frac{\partial j}{\partial y}. \tag{8}$$

The justification to use these simplified equations for analysis of condensing flow is not easy to give rigorously. Comparison with experimental results, for instance by Al-Diwany and Rose [19] and Renz and Jones [6], indicate the model to be not greatly in error, at least.

For laminar flow, the following expressions are used for the fluxes

$$\tau = -\mu\frac{\partial u}{\partial y} \tag{9}$$

$$j = -\frac{\mu}{Sc}\frac{\partial y_1}{\partial y} + \frac{\mu}{Sc}\alpha_T y_1(1 - y_1)\frac{\partial(\ln T)}{\partial y} \tag{10}$$

$$q = -\frac{\mu c_p}{Pr}\frac{\partial T}{\partial y} - \alpha_T RT\frac{\tilde{M}}{\tilde{M}_1 \cdot \tilde{M}_2} j. \tag{11}$$

The system of equations is subject to the following boundary and coupling conditions.

$$(y_L = 0):\ u_L = 0;\quad T_L = T_w;\quad \frac{\partial x_1}{\partial y} = 0 \tag{12}$$

$$(y \to \infty):\ u \to u_\infty;\quad T \to T_\infty;\quad y_1 \to y_{1\infty} \tag{13}$$

$$(y_L = \delta_L;\ y = 0):\ u_L = u \tag{14}$$

$$T_L = T \tag{15}$$

$$\tau_L = \tau \tag{16}$$

$$\dot{m}_L = \dot{m} \tag{17}$$

$$\dot{m}_L = \frac{j}{x_1 - y_1} \tag{18}$$

$$q_L = -\lambda\frac{\partial T}{\partial y} - \alpha_T RT\frac{\tilde{M}}{\tilde{M}_1\tilde{M}_2} j + \dot{m}_L \Delta h_v \tag{19}$$

$$x_1 = f_1(y_1) = f_2(T). \tag{20}$$

The local condensing mass flow density is given by

$$\dot{m} = \dot{m}_L = -\frac{\mathrm{d}}{\mathrm{d}x_L}\int_0^{\delta_L} \rho_L u_L \,\mathrm{d}y_L. \tag{21}$$

The total mass flux for 1 m width of the plate is

$$\dot{m}_{\text{tot}} = \int_0^L \dot{m}\,\mathrm{d}x_L = -\int_0^{\delta_L} \rho_L u_L \,\mathrm{d}y_L. \tag{22}$$

The thermophysical properties of the pure liquids were taken from the tables of the VDI-Wärmeatlas [19] and

Landolt-Börnstein [20]. Simple mixture rules were applied to generate properties of liquid mixtures. The properties of the vapour were calculated according to kinetic theory and ideal gas behaviour.

TRANSFORMATION

For the considered system of partial differential equations a transformation from the physical coordinates x and y to the dimensionless coordinates ξ and η, defined by

$$\xi = \frac{g^* x}{u_\infty^2}, \quad \xi_L = \frac{g^* x_L}{u_\infty^2} \tag{23}$$

$$\eta = \left(\frac{u_\infty}{v_r x}\right)^{\frac{1}{2}} \int_0^y \left(\frac{\rho}{\rho_r}\right) \mathrm{d}y, \quad \eta_L = \left(\frac{u_\infty}{v_{Lr} x_L}\right)^{\frac{1}{2}} y_L \tag{24}$$

is useful. Defining dimensionless streamfunctions, temperatures and concentrations a system of equations and boundary conditions results which has a useful property for reducing computational effort. The equations will not be given here, they can be found in [21]. It is found, as a consequence of this transformation, that the solution to the problem does not depend on the distance x, the velocity u_∞ and the gravity g^* separately, but instead on a combination ξ of these quantities. For the specific case of pure condensing vapour, this combination of variables, a local inverse Froude-number, was already used by Jacobs [22]. A similar combination for the case of combined forced and free convection without change of phase is known since a long time [23]. With the help of ξ the total solution may be structured in three regions, mathematically as well as physically. For $\xi = 0$, the dependence on ξ vanishes leading to a system of ordinary differential equations. Physically, $\xi = 0$ means a vanishing influence of gravity, for instance but not necessarily $g = 0$. For $x \to 0$, i.e. towards the leading edge of the plate, ξ becomes zero, meaning that in this case one has always forced convection, even for high gravity and low free stream velocity. Physically this may be explained from the fact that gravity is a body force while viscous shear leads to an area force. For $x \to 0$ the influence of gravity therefore approaches zero with x^3, while that of shear force does with x^2. For $\xi \to \infty$, again the influence of ξ vanishes and a system of ordinary differential equations is obtained for the limiting case of pure body force convection. For finite values of ξ, no rigorous reduction of the problem to ordinary differential equations is possible, the solution is nonsimilar.

METHODS OF SOLUTION

Exact solutions of the partial differential equations describing the vapour side of the problem may be obtained by finite difference methods. In this work, as in the work of the two other investigators before [6, 13], the procedure of Pantankar and Spalding [24] was used. The system of ordinary differential equations for $\xi = 0$ provided reasonable starting profiles at the leading edge of the plate. This way to start the forward marching calculation appears to be preferable to the choices made in the works cited above. The results in the vapour flow could be inserted into the liquid side equations which allowed an analytical solution due to the use of Nusselt's simplifications. Several different methods were investigated in the numerical procedure, which was complicated considerably by the presence of two coupled fluid phases. In a solution without iteration the liquid layer was solved by using the results of the vapour layer upstream. Damping devices were necessary to obtain stable results with this procedure in some cases, especially for the horizontal plate which was treated in test calculations. Solutions with iteration at each step have been described by Jusionis [13] and Jones and Renz [6]. They were investigated and modified in the present work. Generally speaking, iteration helped to stabilize the numerical procedure, such that no damping devices were necessary in this case. Results for the horizontal plate using the finite difference technique were compared to exact solutions of the ordinary differential equations describing this case. The differences were always less than 1%, supporting confidence in the numerical procedure.

Because of the inherent and well known difficulties encountered in applying finite difference methods to the solution of practical problems by practical engineers, an integral treatment of the problem was investigated in some detail. Encouraged by comparison with the complete solutions, the integral formulation was given for negligible effect of thermodynamic coupling and constant vapour properties evaluated at a suitable reference state. The temperature profile was assumed to be identical to the species concentration profile, an approximation which is reasonable, as heat transfer in the vapour does not have a strong influence on the results anyhow. The energy equation is therefore not needed any more.

The integral equations for momentum and species concentration are then given by

$$u_\infty \frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^\delta u \,\mathrm{d}y\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^\delta u^2 \,\mathrm{d}y\right) - v_i(u_\infty - u_i) = v \left.\frac{\partial u}{\partial y}\right|_{Y=0} - g^* \int_0^\delta \left(1 - \frac{\rho_\infty}{\rho}\right) \mathrm{d}y \tag{25}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^{\delta_{y_1}} u(y_1 - y_{1\infty})\,\mathrm{d}y\right) + v_i(y_{1\infty} - y_{1i}) = -D \left.\frac{\partial y_1}{\partial y}\right|_{y=0}. \tag{26}$$

For the species concentration as well as temperature profile, the following expression was selected

$$\frac{y_1 - y_{1i}}{y_{1i} - y_{1\infty}} = -2\left(\frac{y}{\delta_{y_1}}\right) + \left(\frac{y}{\delta_{y_1}}\right)^2 = \frac{T - T_i}{T_i - T_\infty} \tag{27}$$

which satisfies obvious boundary conditions.

For the velocity profile, two expressions were investigated.

$$\left(\frac{u - u_i}{u_i - u_\infty}\right)_{\mathrm{I}} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2 \tag{28}$$

and

$$\left(\frac{u - u_i}{u_i - u_\infty}\right)_{\mathrm{II}} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2 + \frac{\tilde{u} - u_\infty}{u_i - u_\infty}\left(\frac{y}{\delta}\right)\left[1 - \frac{y}{\delta}\right]. \tag{29}$$

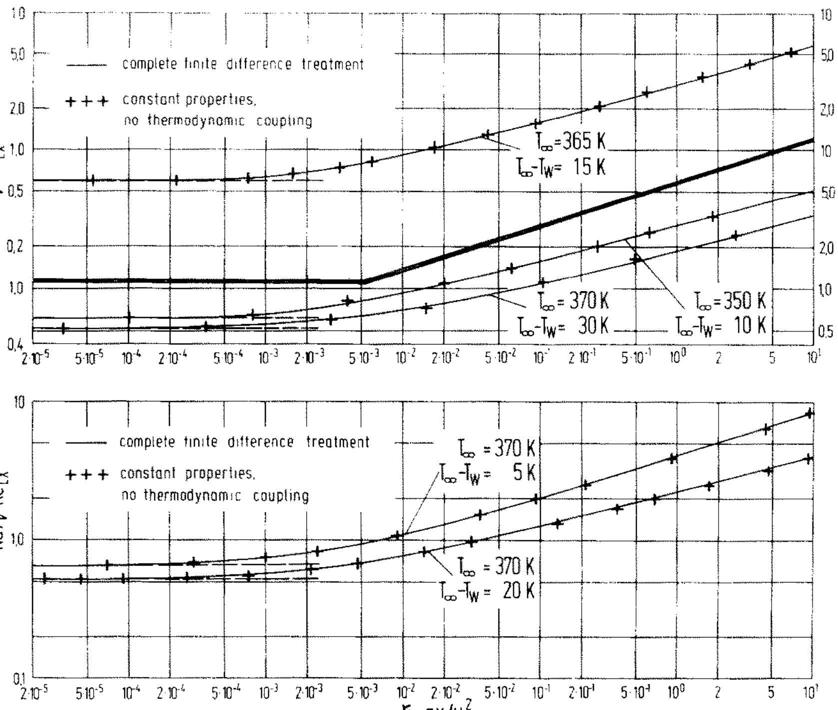

FIG. 2. Modified Nusselt number for specified conditions. Finite difference treatment.

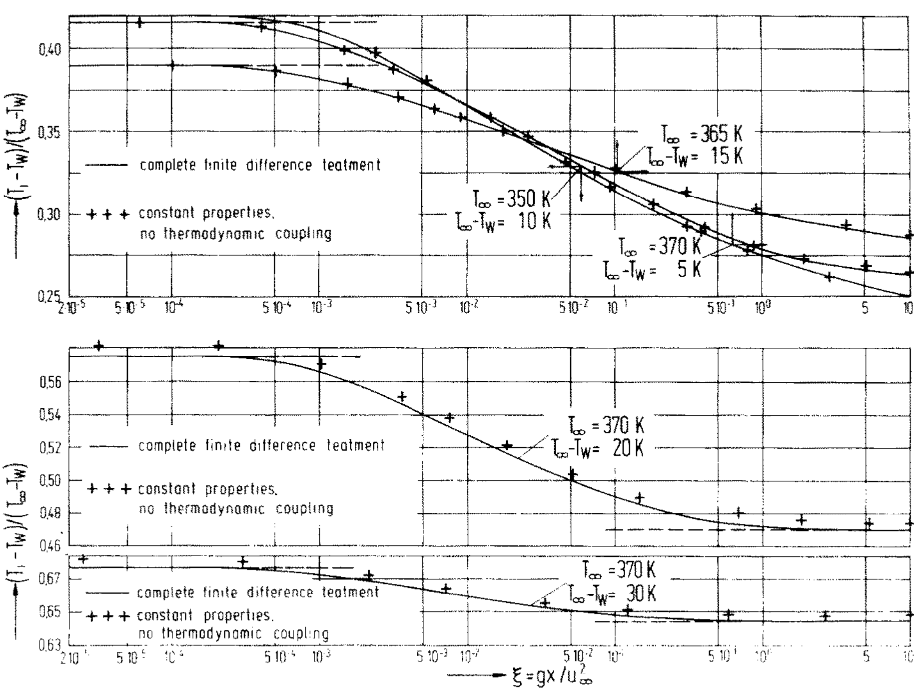

FIG. 3. Reduced temperature difference for specified conditions. Finite difference treatment.

Both expressions satisfy obvious boundary conditions. Neither profile actually contains the influence of suction explicitly. This is however taken into account by δ which decreases with increased suction giving rise to enlarged velocity gradients at the film surface. Profile II is considerably more complicated than profile I. Its main advantage must be seen in the fact that it is capable of an inflection point depending on the value of the quantity $\tilde{u}$ which has the dimension of a velocity. This quantity $\tilde{u}$ was introduced by Rose [25] in his investigation on the limiting case of pure body force condensation. Indeed has such an inflection point been found to exist for small temperature differences $T_\infty - T_w$ [9] in free convection condensation. Its importance in combined forced and free convection is investigated in this work. Both formulations were evaluated and will be discussed below. The full equations will not be given here, since the calculations are basically simple yet lengthy. It should be remarked however, that in case II, it was necessary to assume identical boundary layer thicknesses for velocity and species concentration in order not to have more unknowns than equations. Both integral formulations led to three simple first order ordinary differential equations, which could be solved by standard techniques. To start the calculation, solutions had to be found near the leading edge of the plate. Again this was achieved by considering the case of $\xi = 0$, for which the integral formulations yield algebraic equations which are easily solved. Iteration was again necessary at each step if excessively small forward steps were to be avoided in order to save computer time.

RESULTS AND DISCUSSION

In order to evaluate the heat flux density the Nusselt number Nu and the temperature T_i at the interface must be known. They will be given in this paper separately for some specific cases of methanol–water condensation. Some general results for the case of a horizontal plate may be found in another paper of the same author [26].

In Figs. 2 and 3 some solutions obtained with the finite difference method are shown. In Fig. 2 the modified Nu number $Nu/\sqrt{(Re_{Lx})}$ is plotted against ξ, in Fig. 3 the analogous presentation is given for the reduced temperature difference $T_i - T_w/T_\infty - T_w$. In both figures, the structure of the solution as a function of ξ is clearly recognized. For low values of ξ, the limiting case of forced convection, the modified Nu number as well as the temperature T_i are independent of ξ. For high values of ξ, towards the limiting case of free convection, the temperature difference becomes again independent of ξ. Its value is lower than in the case $\xi = 0$. The Nu-number approaches the functional dependence $\xi^{1/4}$. This is in agreement with Nusselt's theory for negligible vapour flow and is true rigorously even if the body force convection occurring in the vapour mixture is taken into account. The limiting cases of forced and free convection are reached rigorously only for $\xi = 0$ resp. $\xi = \infty$, the latter not being presentable in the figures. One can see, however, that even for finite values of ξ the limiting cases are reached with sufficient accuracy. This fact of course has a considerable practical importance since the limiting cases can be calculated from ordinary differential equations. Unfortunately no clear *a priori* criterion can be given, as the phenomena depend not only on ξ but also on the specific condensation conditions. If no high accuracy is required, one can deduce critical values of ξ for practical purposes. Definitely the usual practice of neglecting one effect or the other depending on values of the velocity u_∞ is not justified from a rigorous standpoint, though certainly useful in order of magnitude estimations. Also represented in the figures are results without thermodynamic coupling and with constant vapour properties evaluated at a reference state. Thermal diffusion and diffusional thermo are completely negligible for the considered case, which does not exclude that they become more important for other mixtures, especially those with very different molecular weights. In the constant property analysis, a suitable reference state was found to be

$$T_r = \tfrac{1}{2}(T_i + T_\infty) \tag{30}$$

with the species concentration of the saturated vapour at Tr. This simplified analysis approximates the complete solution to within 2% and is therefore quite satisfactory. This result, of course, rests entirely on the fact that small temperature differences in a vapour were investigated here, such that the property variation over the vapour layer should not be very large. This however is a case quite frequently encountered in condensation phenomena.

Figures 4 and 5 show the agreement between the finite difference treatment and both integral formulations. The agreement is entirely satisfactory in the whole region represented in the figures, where the two integral formulations do not differ at all to any significant extent. This result is quite important, and plausible, too. Condensation phenomena, at least in the region considered in the figures, are characterized by simple profiles of temperature, species concentration and velocity in the vapour layer. They can be approximated with good accuracy by the simple parabolic expressions of integral formulation I. Differences however show up in some cases when the limiting case of free convection is approached closer. Exact solutions for body force condensation of methanol–water mixtures have been presented earlier [9, 11]. Comparing these free convection results of Tamir and Taitel [11] for the heat flux with those of integral formulation I leads to excellent agreement for high condensation rates, i.e. high values of the temperature difference $T_i - T_\infty$. For small condensation rates the agreement becomes poor. Typical examples are deviations of 15% for $T_\infty = 365$ K, $T_w = 350$ K and more than 50% for $T_\infty = 370$ K, $T_w = 365$ K. These deviations cannot of course be explained by differences in the fluid properties, though these may add to the picture. Instead the origin of the discrepancies was verified to lie in the more complicated velocity field for the low condensation rates, where inflection points had to be taken into account. This is impossible for integral formulation I, as the expression for the velocity profile there is a simple parabola. Integral formulation II however admits such an inflection point depending on the quantity $\tilde{u}$. Figure 6 substantiates this statement. The velocity ratio u/u_∞ as calculated from integral formulation II is plotted against the dimensionless boundary-layer thickness y/δ for several values of ξ measuring the influence of free convection. The conditions are $T_\infty = 370$ K and $T_w = 365$ K. It can clearly be seen that up to $\xi = 2.6$ no inflection point shows up in the velocity profile, yet does at a value of $\xi = 15$. For conditions of higher condensation rate, i.e. $T_\infty = 370$ K and $T_w = 350$ K as shown in Fig. 7, no inflection point is found up to values of ξ as high as 87, where the limit of body force convection may be assumed to be attained. This

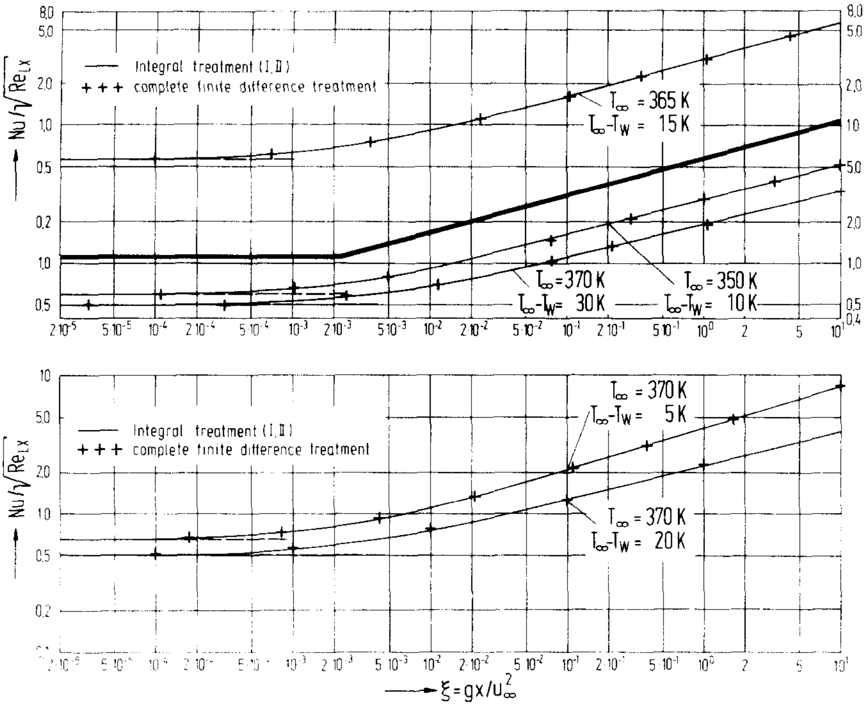

FIG. 4. Comparison of integral and finite difference treatment for modified Nusselt number.

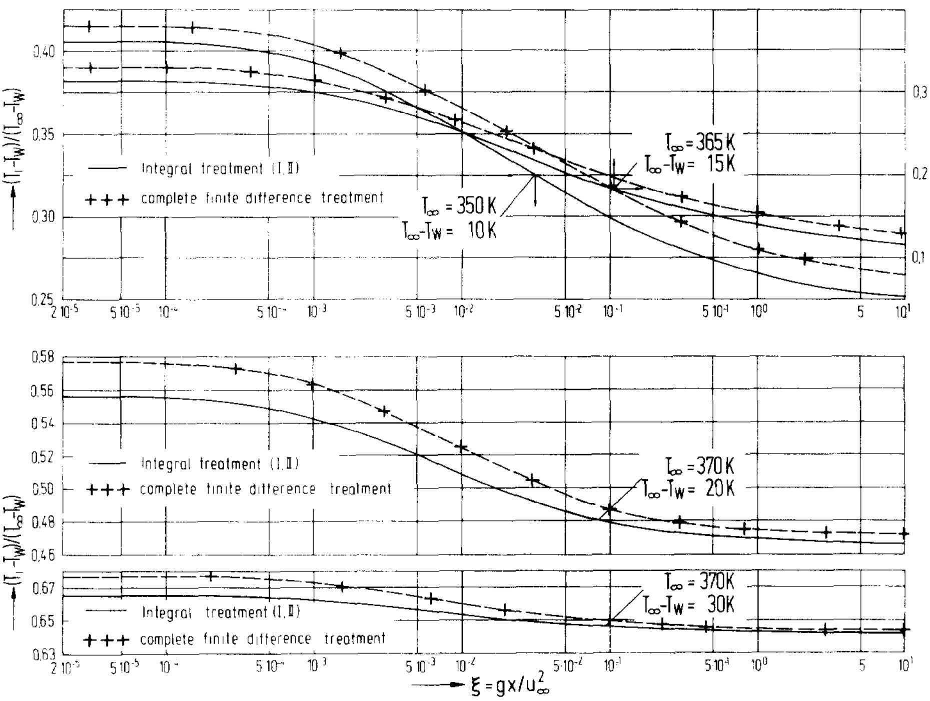

FIG. 5. Comparison of integral and finite difference treatment for reduced temperature difference.

explains why both integral formulations do not differ within the region considered in the figures, yet the first gives poor results for low condensation rates and good results for high condensation rates in the limiting case of free convection. As may be expected from this investigation, the results of integral formulation II are in much better agreement with the low condensation rate results of Tamir and Taitel [11]. For instance, in the examples cited above the agreement is almost perfect for the case $T_\infty = 365\,K$, $T_w = 350\,K$, which may be fortuitious. For $T_\infty = 370\,K$ and $T_w = 365\,K$ agreement is within 18% which is by far the worst case encountered. Thus the effort of applying a more complicated expression for the velocity profile is justified and necessary only for cases of low condensation rate and dominant influence of body force.

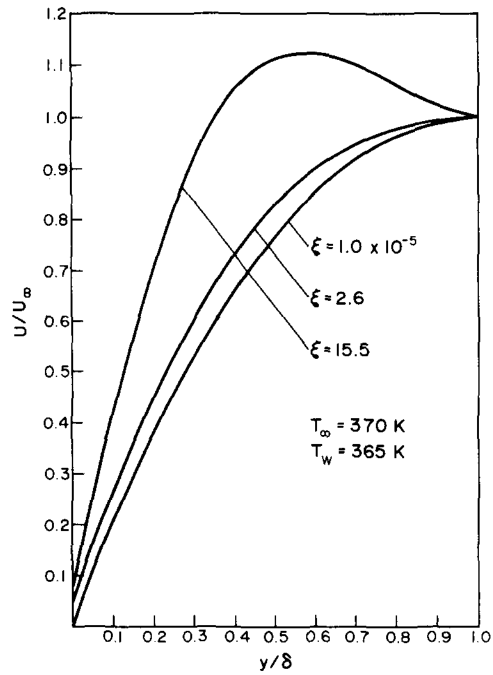

FIG. 6. Velocity profiles in the vapour at low condensation rates.

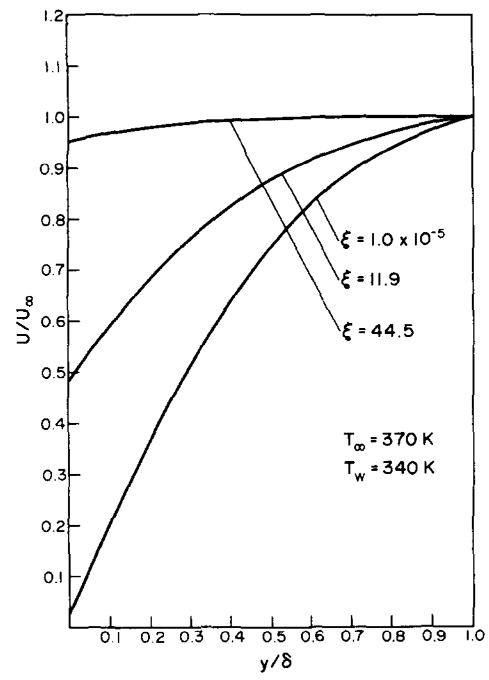

FIG. 7. Velocity profiles in the vapour at high condensation rates.

Generally speaking the whole region of condensation conditions investigated may well be analyzed with sufficient accuracy by the approximate integral technique. The computer time saved in comparison with the finite difference calculation is considerable, the time necessary for the integral treatment being about one tenth of that in the finite difference treatment.

Acknowledgement—The author wishes to acknowledge the assistance of K. Bolst, M. Buse, K. Homann and M. Wegmann during the numerical calculations. He also is indebted to U. Renz for his cooperation in connection with his version of the Patankar–Spalding computer program. He finally wishes to thank Prof. Dr. K. Stephan, the director of the institute, for his interest, encouragement and helpful remarks.

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CONDENSATION LAMINAIRE EN FILM DE MELANGES DE VAPEUR EN PRESENCE DE FORCES VOLUMIQUES EN CONVECTION FORCEE—METHODES INTEGRALE ET DE DIFFERENCES FINIES

Résumé—On considère la condensation laminaire en film d'une vapeur binaire en présence de forces volumiques en convection forcée sur une plaque plane. Après un changement convenable de coordonnées on constate que la solution générale s'exprime à l'aide d'une longueur ξ rendue adimensionnelle. Le système d'équations est résolu à l'aide d'une méthode de différences finies et de deux méthodes intégrales. Les deux traitements intégraux différent l'un de l'autre par l'expression du profil de vitesse, l'un d'entre eux admettant un point d'inflexion. Cette expression plus compliquée pour le profil de vitesse est importante pour le traitement du cas d'un faible taux de condensation avec prédominance des forces de volume. De manière générale, le traitement intégral est très satisfaisant. On discute l'influence du transfert thermique dans la vapeur, des variations des propriétés du fluide et des couplages thermodynamiques.

LAMINARE FILMKONDENSATION VON GEMISCHDÄMPFEN AN DER PLATTE BEI ÜBERLAGERUNG VON ERZWUNGENER UND FREIER STRÖMUNG—INTEGRAL- UND DIFFERENZENVERFAHREN

Zusammenfassung—Es wird die laminare Filmkondensation eines binären Dampfgemisches an einer ebenen Platte bei Überlagerung von erzwungener und freier Strömung untersucht. Mit Hilfe einer geeigneten Koordinatentransformation läßt sich das gesamte Lösungsgebiet des Gleichungssystems nach Werten der dimensionslosen Lauflänge ξ in mehrere Teilbereiche gliedern. Das Gleichungssystem wird mit einem Differenzenverfahren und zwei Integralverfahren gelöst. Beide Integralverfahren unterscheiden sich durch den Ansatz für das Geschwindigkeitsprofil, wobei beim einen ein Wendepunkt zugelassen ist, beim anderen nicht. Es zeigt sich, daß der Wendepunkt nötig ist, wenn Fälle niedriger Kondensationsrate und dominierendem Schwerkrafteinfluß untersucht werden. Insgesamt ist die Leistungsfähigkeit des Integralverfahrens sehr befriedigend. Untersucht werden außerdem der Einfluß der Wärmeleitung im Dampf, variabler Stoffwerte sowie der thermodynamischen Kopplung durch Thermodiffusion und Diffusionsthermik.

ИССЛЕДОВАНИЕ СОВМЕСТНОЙ ЕСТЕСТВЕННОЙ И ВУНУЖДЕННОЙ КОНВЕКЦИИ ПРИ ЛАМИНАРНОЙ ПЛЕНОЧНОЙ КОНДЕНСАЦИИ БИНАРНЫХ ПАРОВ С ПОМОЩЬЮ ИНТЕГРАЛЬНОГО И КОНЕЧНО-РАЗНОСТНОГО МЕТОДОВ

Аннотация — Рассматриваются ламинарная пленочная конденсация при совместной естественной и вынужденной конвекции бинарного пара на плоской пластине. Показано, что после соответствующего преобразования координат общее решение строится по безразмерной длине ξ. Система уравнений решается конечно-разностным методом и двумя интегральными методами, которые отличаются друг от друга наличием выражения для профиля скорости, причем в одном из них допускается точка перегиба. Это более сложное выражение для профиля скорости важно в случае рассмотрения малой скорости конденсации и доминирующего влияния массовой силы. Вообще говоря, интегральный метод весьма удовлетворителен. Обсуждается влияние переноса тепла при наличии термической диффузии и термодиффузии на свойства пара, жидкости и термодинамическое взаимодействие.